Generalised Local Search Machines

Holger H. Hoos & Thomas Stützle
Outline

1. The Basic GLSM Model
2. State, Transition and Machine Types
3. Modelling SLS Methods Using GLSMs
4. Extensions of the Basic GLSM Model
The Basic GLSM Model

Many high-performance SLS methods are based on combinations of simple (pure) search strategies (e.g., ILS, MA).

These hybrid SLS methods operate on two levels:

- **lower level**: execution of underlying simple search strategies
- **higher level**: activation of and transition between lower-level search strategies.

Key idea underlying Generalised Local Search Machines: Explicitly represent higher-level search control mechanism in the form of a finite state machine.
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Example: Simple 3-state GLSM

States $z_0$, $z_1$, $z_2$ represent simple search strategies, such as Random Picking (for initialisation), Iterative Best Improvement and Uninformed Random Walk.

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- States \( \cong \) simple search strategies.
- State transitions \( \cong \) search control.

- GLSM \( M \) starts in initial state.

- In each iteration:
  - \( M \) executes one search step associated with its current state \( z \);
  - \( M \) selects a new state (which may be the same as \( z \)) in a nondeterministic manner.

- \( M \) terminates when a given termination criterion is satisfied.
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Formal definition of a GLSM

A Generalised Local Search Machine is defined as a tuple $\mathcal{M} := (Z, z_0, M, m_0, \Delta, \sigma_Z, \sigma_\Delta, \tau_Z, \tau_\Delta)$ where:

- $Z$ is a set of states;
- $z_0 \in Z$ is the initial state;
- $M$ is a set of memory states (as in SLS definition);
- $m_0$ is the initial memory state (as in SLS definition);
- $\Delta \subseteq Z \times Z$ is the transition relation;
- $\sigma_Z$ and $\sigma_\Delta$ are sets of state types and transition types;
- $\tau_Z : Z \mapsto \sigma_Z$ and $\tau_\Delta : \Delta \mapsto \sigma_\Delta$ associate every state $z$ and transition $(z, z')$ with a state type $\sigma_Z(z)$ and transition type $\tau_\Delta((z, z'))$, respectively.
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Example: Simple 3-state GLSM (formal definition)

- $Z := \{z_0, z_1, z_2\}; \ z_0 = \text{initial machine state}$
- no memory ($M := \{m_0\}; \ m_0 = \text{initial and only memory state}$)
- $\Delta := \{(z_0, z_1), (z_1, z_2), (z_1, z_1), (z_2, z_1), (z_2, z_2)\}$
- $\sigma_Z := \{z_0, z_1, z_2\}$
- $\sigma_\Delta := \{\text{PROB}(p) \mid p \in \{1, p_1, p_2, 1 - p_1, 1 - p_2\}\}$
- $\tau_Z(z_i) := z_i, \ i \in \{0, 1, 2\}$
- $\tau_\Delta((z_0, z_1)) := \text{PROB}(1), \ \tau_\Delta((z_1, z_2)) := \text{PROB}(p_1), \ldots$
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  - With probability 1, switch to state $z_1$.
  - Perform one search step according to state $z_1$; switch to state $z_2$ with probability $p_1$, otherwise, remain in state $z_1$.
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Transition types formally represent mechanisms used for switching between GLSM states.

Multiple states / transitions can have the same type.

$\sigma_Z, \sigma_\Delta$ should include only state and transition types that are actually used in given GLSM (‘no junk’).

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GLSM Semantics

Behaviour of a GLSM is specified by machine definition + run-time environment comprising specifications of

- state types,
- transition types;
- problem instance to be solved,
- search space,
- solution set,
- neighbourhood relations for subsidiary SLS algorithms;
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Run GLSM $M$:

set \textit{current machine state} to $z_0$; set \textit{current memory state} to $m_0$;

While \textit{termination criterion} is not satisfied:

- perform \textit{search step} according to type of current machine state; this results in a new \textit{search position};

- select \textit{new machine state} according to \textit{types of transitions} from \textit{current machine state}, possibly depending on \textit{search position} and \textit{current memory state}; this may change the \textit{current memory state}.
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- The *current search position* is only changed by the subsidiary search strategies associated with states, *not* as side-effect of machine state transitions.
- The *machine state* and *memory state* are only changed by state-transitions, *not* as side-effect of search steps. (Memory state is viewed as part of higher-level search control.)
- The operation of $\mathcal{M}$ is uniquely characterised by the evolution of *machine state*, *memory state* and *search position* over time.
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GLSMs are factored representations of SLS strategies:

- Given GLSM represents the way in which *initialisation* and *step function* of a hybrid SLS method are composed from respective functions of *subsidiary component SLS methods*.

- When modelling hybrid SLS methods using GLSMs, *subsidiary SLS methods* should be as simple and pure as possible, leaving *search control* to be represented explicitly at the GLSM level.

- *Initialisation* is modelled using *GLSM states* (advantage: simplicity and uniformity of model).

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- the GLSM model (states, transitions, ...);
- the search method associated with each state type, i.e., step functions for the respective subsidiary SLS methods;
- the semantics of each transition type, i.e., under which conditions respective transitions are executed, and how they effect the memory state.
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State types

- State type semantics are often most conveniently specified procedurally (see algorithm outlines for ‘simple SLS methods’ from Chapter 2).

- *initialising state type* = state type \( \tau \) for which search position after one \( \tau \) step is independent of search position before step.

\[ \text{initialising state} = \text{state of initialising type}. \]

- *parametric state type* = state type \( \tau \) whose semantics depends on memory state.

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Transitions types (1)

- **Unconditional deterministic transitions** – type *DET*:
  - executed always and independently of memory state or search position;
  - every GLSM state can have at most one outgoing DET transition;
  - frequently used for leaving initialising states.

- **Conditional probabilistic transitions** – type *PROB(p)*:
  - executed with probability $p$, independently of memory state or search position;
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  - executed always and independently of memory state or search position;
  - every GLSM state can have at most one outgoing DET transition;
  - frequently used for leaving initialising states.

- **Conditional probabilistic transitions** – type \(\text{PROB}(p)\):
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- DET transitions are a special case of PROB transitions.

- For a GLSM $\mathcal{M}$ any state that can be reached from initial state $z_0$ by following a chain of PROB($p$) transitions with $p > 0$ will eventually be reached with arbitrarily high probability in any sufficiently long run of $\mathcal{M}$.

- In any state $z$ with a PROB($p$) self-transition $(z, z)$ with $p > 0$, the number of GLSM steps before leaving $z$ is distributed geometrically with mean and variance $1/p$. 
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Transitions types (2)

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  - all CPROB transitions from the current GLSM state whose condition predicates are not satisfied are *blocked*, i.e., cannot be executed.

Note:

- Special cases of CPROB\((C, p)\) transitions:
  - \( \text{PROB}(p) \) transitions;
  - *conditional deterministic transitions*, type \( \text{CDET}(C) \).

- Condition predicates should be efficiently computable (ideally: \( \leq \) linear time w.r.t. size of given problem instance).
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\[ \text{count}(k) \] total number of GLSM steps \( \geq k \)
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\[ \text{scount}(k) \] number of GLSM steps in current state \( \geq k \)
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\( \text{lmin} \) current candidate solution is a local minimum w.r.t. the given neighbourhood relation

\[ \text{evalf}(y) \] current evaluation function value \( \leq y \)

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All based on local information; can also be used in negated form.
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- Associated with individual transitions; provide mechanism for modifying current memory states.
  - Performed whenever GLSM executes respective transition.
  - Modify memory state only, *cannot* modify GLSM state or search position.
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Machine types:

Capture *structure of search control mechanism*, obtained by abstracting from state and transition types of GLSMs.

- **1-state machines:**
  - simplest machine type, single initialising state only;
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  - one initialising + one working state;
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  ![Diagram of sequential 1-state machine with states Z0 and Z1]

  - visit initialising state $z_0$ only once.

- **alternating 1-state+init machines:**

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- one initialising state (visited only once), two working states;

- any search trajectory can be partitioned into three phases: one initialisation step, a sequence of $z_1$ steps and a sequence of $z_2$ steps.
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Generalisations:

▶ **k-state+init sequential machines:**
  - one initialising state (visited only once), \( k \) working states;
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Modelling SLS Methods Using GLSMs

Uninformed Picking and Uninformed Random Walk

**procedure** step-RP$(\pi, s)$

*input:* problem instance $\pi \in \Pi$, candidate solution $s \in S(\pi)$

*output:* candidate solution $s \in S(\pi)$

$s' := \text{selectRandom}(S)$;

return $s'$

**end** step-RP

**procedure** step-RW$(\pi, s)$

*input:* problem instance $\pi \in \Pi$, candidate solution $s \in S(\pi)$

*output:* candidate solution $s \in S(\pi)$

$s' := \text{selectRandom}(N(s))$;

return $s'$

**end** step-RW
Uninformed Random Walk with Random Restart

\[ R = \text{restart predicate, e.g., countm}(k) \]
Iterative Best Improvement with Random Restart

\begin{algorithm}
\begin{algorithmic}
\Procedure{step-BI}{$\pi, s$}
\Statex \textbf{input:} problem instance $\pi \in \Pi$, candidate solution $s \in S(\pi)$
\Statex \textbf{output:} candidate solution $s \in S(\pi)$
\State $g^* := \min\{g(s') \mid s' \in N(s)\}$;
\State $s' := \text{selectRandom}\{s' \in N(s) \mid g(s') = g^*\}$;
\State \Return $s'$
\EndProcedure
\end{algorithmic}
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Randomised Iterative Best Improvement with Random Restart

\[
\begin{align*}
\text{RP} & \xrightarrow{\text{PROB}(1-p)} \text{BI} \\
\text{BI} & \xrightarrow{\text{CDET}(R)} \text{RP} \\
\text{RP} & \xrightarrow{\text{CDET}(R)} \text{RW} \\
\text{RW} & \xrightarrow{\text{CDET}(R)} \text{BI} \\
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\end{align*}
\]

\[
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\text{CPROB}(\text{not } R, 1-p) & \\
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Simulated Annealing

- Note the use of transition actions and memory for temperature $T$.
- The parametric state $SA(T)$ implements probabilistic improvement steps for given temperature $T$.
- The initial temperature $T_0$ and function $update$ implement the annealing schedule.
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Iterated Local Search (1)

- The acceptance criterion is modelled as a state type, since it affects the search position.
- Note the use of transition actions for memorising the current candidate solution (pos) at the end of each local search phase.
- Condition predicates $CP$ and $CL$ determine the end of perturbation and local search phases, respectively; in many ILS algorithms, $CL := l\text{min}$.
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Iterated Local Search (2)

\begin{equation}
\text{procedure}\ step-AC(\pi, s, t) \\
\text{input: problem instance } \pi \in \Pi, \\
\text{candidate solution } s \in S(\pi) \\
\text{output: candidate solution } s \in S(\pi) \\
\text{if } C(\pi, s, t) \text{ then} \\
\text{\hspace{0.5cm} return } s \\
\text{else} \\
\text{\hspace{0.5cm} return } t \\
\text{end} \\
\text{end step-AC}
\end{equation}
Ant Colony Optimisation (1)

- General approach for modelling population-based SLS methods, such as ACO, as GLSMs:

  Define search positions as *sets of candidate solutions*; search steps manipulate some or all elements of these sets.

  *Example:* In this view, Iterative Improvement (II) applied to a population $sp$ in each step performs one II step on each candidate solution from $sp$ that is not already a local minimum.

  (Alternative approaches exist.)

- Pheromone levels are represented by memory states and are initialised and updated by means of transition actions.
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Extensions of the Basic GLSM Model

The basic GLSM model can be generalised and extended in various rather straightforward ways, such as:

- Co-operative GLSM models
- Learning GLSM models
- Evolutionary GLSM models
- Continuous GLSM models

Note: So far, these extensions remain mostly unexplored — lots of opportunities for interesting research!
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» **Key idea:** Apply multiple GLSMs simultaneously to the same problem instance

» Naturally captures population-based SLS approaches.

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- GLSMs in population exchange information about their search trajectories, e.g., via message passing or blackboard mechanism.
- Communication can be modelled via shared memory state or special transition actions (e.g., send, receive).
- These models are naturally suited for representing population-based algorithms that use communication between individual search agents, such as ACO.
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- **Key idea:** In a GLSM with probabilistic transitions, let transition probabilities evolve over time to adaptively optimise search control strategy.

- Can build on concepts from learning automata theory.

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- Further extensions:
  - support mutation / recombination operations on GLSMs;
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- The main feature of the GLSM model, namely its clear distinction between *lower-level, simple search strategies* and *higher-level search control*, equally applies to continuous SLS algorithms.

- **Key idea:** Model complex continuous SLS methods by using continuous optimisation procedures as subsidiary local search strategies.

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